The Derivatives Technology Foundation

Project Proposal

"American Option Pricing in the Heston Model"

Project Description

The Heston Model for stochastic volatility option pricing models [1] is given by

$$dS_t = \mu S_t dt + S_t \sqrt{\sigma_t} dW_t$$
$$d\sigma_t = \kappa(\theta - \sigma_t)dt + \omega\sqrt{\sigma_t}\left(\rho dW_t + \sqrt{1-\rho^2} d\hat{W}_t\right)$$

where the parameters are assumed to be constant and known and the two Brownian Motions are assumed to be independent. The model is very popular among practitioners since it is possible to derive more or less explicit closed-form solutions (in the form of an integral that needs to be calculated numerically) for European call and put options in the absence of dividends and with fixed interest rates.

Options with have the possibility of early exercise (American options) and options that involve the payment of cash dividends at at predetermined dates during the lifetime of the option, no longer admit such closed form solutions. Since quick calculation of American option prices in the presence of dividends is essential for realistic applications, other numerical methods have been developed, which are usually based on solving the Heston partial differential equation for American options.

Examples of this approach include Clarke and Parrott [2] who treat this as a linear complimentarity problem and use an implicit finite difference scheme combined with a multigrid procedure. Their method was improved significantly by Oosterlee [3] In that paper the smoothing procedure in the multigrid procedure is chosen in an optimal way after an extensive analysis of numerical errors. Forsyth, Vetzal and Zvan have used a penalty method to deal with the early exercise constraint [4], and Ikonen & Toivanen [5] use an operator splitting method.

Recently an alternative method has been designed which is based on building a discrete time process (a 'tree') that approximates the dynamics of the Heston model. in the paper by Leisen [6] a tree is designed with a proof of weak convergence to the correct distribution of asset prices. Florescu and Viens [7] suggest that the proof is incorrect, but their alternative [8] involves a tree with Monte Carlo simulations at every time step, which is therefore expected to be much too slow for practical purposes.

These tree methods all use a grid with grid points at distance of order $\sqrt{\Delta t}$, where Δt denotes the discrete time step. Recently, it was proposed [9] to use as tree in which we define a fixed grid for the logarithm of the stock and the squared volatility process. We take grids that change every timestep but with a meshsize which is of order $o(\Delta t)$ at every timestep. The discrete time stochastic process that we define takes its values on this grid, and has 16 successor nodes in every point. This obviously means that we require more computation time per timestep. But the extra degrees of freedom that we create by this setup allows us to exploit the fact that the American Option price function is once continuously differentiable in both state variables. This smoothness is used to improve the speed of convergence, which means that we require more computations per timestep but far less timesteps than in other methods.

. We would like to investigate:

- **Theoretical Part**
  How should the methods defined by Oosterlee [3] and Vellekoop/Nieuwenhuis [9] be extended to include the possibility of cash dividends (at predetermined times, with known cash dividend value) ?

- **Practical Part**
    1. How do the speed vs. accuracy characteristics of these two different methods compare when we want to calculate prices and Greeks (i.e. hedging parameters) for American options with cash dividends ?
    2. Are the methods quick and accurate enough to calibrate a Heston model to market option data?

  If time permits, alternative methods such as those of Forsyth et al. [4] and Ikonen et al. [5] could be investigated as well.

Starting Literature

1. Heston, S.L., *A closed form solution for options with stochastic volatilities.* Review of Financial Studies, vol. 6(2), pp. 327-343, 1993.
2. Parrott, K. & Nigel Clarke. Multigrid for American option pricing with stochastic volatility, *Applied Mathematical Finance*, 6(3):177–195, 1999.
3. C.W. Oosterlee. On multigrid for linear complimentarity problems with application to American-style options. *Electronic Transactions on Numerical Analysis*, 15:165–185, 2003.
4. R. Zvan, P. Forsyth, and K. Vetzal. A penalty method for American options with stochastic volatility. *J. Comp. Appl. Math*., pages 199–218, 1998.
5. Ikonen, S. & Toivanen, J., Operator Splitting Methods for Pricing American Options with Stochastic Volatility, *Reports of the Department of Mathematical Information Technology*, University of Jyväskylä, Department of Mathematical Information Technology, 2004.
6. D.P.J. Leisen. Stock evolution under stochastic volatility: A discrete approach. *Journal of Derivatives*, 8:8–27, 2000.
7. I. Florescu and F. Viens. A binomial tree approach to stochastic volatility driven model of the stock price. *An. Univ. Craiova Ser. Mat. Inform.*, 32:126–142, 2005.
8. Florescu and F. Viens. Stochastic volatility: option pricing using a multinomial recombining tree. *Preprint* Department of Statistics, Purdue University, 2006.
9. Vellekoop, M.H. & Nieuwenhuis, J.W., A tree-based Method to price American Options in the Heston Model. *Preprint*, University of Twente, 2006.

Proposed Location for Project
Saen Options

Proposed Duration for Project
6 to 9 months

Supervisor
Michel Vellekoop, TDTF & University of Twente

Co-supervisors
Bastiaan de Geeter, TDTF & Saen Options
Nico vd Hijligenberg, TDTF & SFISS Financial Technology
Feedback will also be sought from the other parties who participate in TDTF.

Michel Vellekoop
The Derivative Technology Foundation.

Version 09-01-07.